An experimental study of frequency regimes of honey coiling

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Abstract

A stream of viscous fluid falling from a fixed height tends to coil with specific behaviors that are influenced by fluid properties, fall height, and flow rate. Previous studies have shown the existence of three distinct regimes of liquid rope coiling: the viscous, inertial, and gravitational regimes, as well as a transition regime between the inertial and gravitational regimes, referred to as the inertio-gravitational (IG) regime. Scaling laws can be used to estimate the radius of the coils and coiling frequency given the fall height and fluid properties. In the IG regime, the coiling frequency of the liquid rope, with all initial parameters fixed, is multi-valued [1]. Jumps between different coiling frequencies in this regime are often accompanied by a change in the sense of rotation of the coil, accomplished through a 'figure of eight' transition [1]. The purpose of this study is to experimentally verify the trends in coiling frequency indicated in the scaling laws and the existence of multiple coiling frequencies within the IG regime.

1 Introduction

In this study, honey with kinematic viscosity ν and density ρ is ejected from an orifice of radius a_0 from a fixed height (H) at an approximately steady volumetric flow rate (Q) onto a flat surface. The radius of the stream immediately above the coil portion of the system is defined as a_1. Previous studies have developed scaling laws that describe the coiling frequency of the viscous rope in the different coiling regimes. In this study, experimental observations are used to verify the existence of the different regimes, the validity of the scaling laws, and the multi-valued nature of the IG regime.

In general, coiling frequency is determined by the balance of viscous (F_V), gravitational (F_G), and inertial (F_I) forces in the coil portion of the viscous rope [2]. Coiling is observed

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in the viscous regime when gravitational and inertial forces are insignificant. Gravitational coiling occurs when viscous forces are approximately balanced by gravitational forces, and inertial forces are negligible by comparison. Coiling is seen in the inertial regime when inertial and viscous forces are approximately balanced, and gravitational forces are negligible. Coiling in the IG regime occurs when viscous forces are balanced by both inertial and gravitational forces [1]. In this regime, the coiling frequency is not uniquely determined for fixed initial parameters, but is multi-valued [1]. Ribe gave a way of estimating the three forces [2],and given the F_V, F_G, and F_I forces per unit length of the liquid rope, it is straightforward to derive scaling laws for coiling frequency.

To achieve steady coiling, there must be a balance of all forces involved. The viscous force acts as resistance to movement. Thus, the viscous force must balance the gravitational and inertial forces, where the inertial force is caused by the buckling of the liquid rope when hitting the surface. Two scaling laws are derived from instances in which the inertial or gravitational force is insignificant, and one scaling law occurs when all forces need to be considered. For this reason, there are three regimes that are considered. From these scaling laws, one can determine that the observed frequencies should be proportional to certain frequencies, depending on the regime.

Experimentally, Ribe, *et.al* [2] showed that the regime of coiling for highly viscous liquids can be determined from the dimensionless height

$$H(g/v^2)^{1/3}. \tag{1}$$

This dimensionless parameter does not, however, dictate the strict dynamics of the coil; it is a general marker for which scaling law to be used. Each experimental trial can be characterized by the dimensionless parameters (1) and

$$\Pi_1 = \left(\frac{\nu^5}{gQ^3}\right)^{1/5}. \tag{2}$$

The relationship between Π_1 and coiling frequency is discussed in more detail later, but Π_1 can be perceived as a non dimensional viscosity.

1.1 Motivation

Figure of eights resulting in a change in the sense of rotation of the coil are present in a data set provided from a 2002 experimental study of honey coiling. However, the figure of eight occurs at a time during the experiment when the flow rate of the viscous stream is not constant. Furthermore, the starting height from which the honey is released, strongly suggests that all coiling in the video should be characteristic of the inertial regime. It is possible that a figure of eight in such an instance is the result of some unexpected instability, possibly initiated by secondary buckling or by the changing flow rate. The goal of this study is to attempt to replicate the figure of eight behavior seen in the 2002 data set, but to do so in a manner that more carefully regulates flow rate. Previous research [1] suggests that changes in the sense of rotation of the coil and in coiling frequency in the IG regime often occur through a figure of eight transition. Scaling laws can help predict the range of heights over which coiling in the IG regime will occur.

1.2 Scaling laws

Previous research has determined, [1],

$$F_V \sim \rho \nu a_1^4 U_1 R^{-4} \tag{3}$$
$$F_G \sim \rho g a_1^2 \tag{4}$$
$$F_I \sim \rho a_1^2 U_1^2 R^{-1}, \tag{5}$$

where a_1 is the radius of the stream within the coil, and $U_1 = \frac{Q}{\pi a_1^2}$ is the axial velocity of the fluid. R is the radius of the coil, measured from the center of the stream, and thus the coiling frequency, Ω, is subject to the relation $\Omega = U_1/R$.

In the gravitational regime, occurring by experimental study when $.08 < H(\frac{g}{\nu^2})^{1/3} < .4$, inertial forces are negligible and gravitational forces are balanced by viscous forces ($F_G \approx F_V >> F_I$). So the scaling laws give us

$$F_G \sim \rho g a_1^2 \approx \rho \nu a_1^4 U_1 R^{-4} \sim F_V. \tag{6}$$

From this, it follows:

$$R \sim g^{-1/4} \nu^{1/4} Q^{1/4} \equiv R_G. \tag{7}$$

In addition,the coiling frequency in this regime is:

$$\Omega \sim g^{1/4} \nu^{-1/4} a_1^{-2} Q^{3/4} \equiv \Omega_G. \tag{8}$$

In the inertial regime, occurring when $H(\frac{g}{\nu^2})^{1/3} > 1.2$, gravitational forces are negligible and viscous forces are balanced by inertial forces ($F_I \approx F_V >> F_G$). So the scaling laws show

$$F_I \sim \rho a_1^2 U_1^2 R^{-1} \approx \rho \nu a_1^4 U_1 R^{-4} \sim F_V. \tag{9}$$

From this, it follows:

$$R \sim \nu^{1/3} a_1^{4/3} Q^{-1/3} \equiv R_I. \tag{10}$$

In addition, the coiling frequency in this regime is:

$$\Omega \sim \nu^{-1/3} a_1^{-10/3} Q^{4/3} \equiv \Omega_I. \tag{11}$$

1.3 Coiling in IG Regime

This regime is a transition between the gravitational and inertial regimes, and is thus important when both the inertial forces and gravitational forces are taken into account. Experimentally, it has been shown that this occurs when $.7 < \frac{\Omega_I}{\Omega_G} < 2$ [3]. Using the explicit forms of Ω_G (8) and Ω_I (11), this implies

$$.7 < \left(\frac{Q^7}{g^3 \nu a_1^{16}}\right)^{1/12} < 2. \tag{12}$$

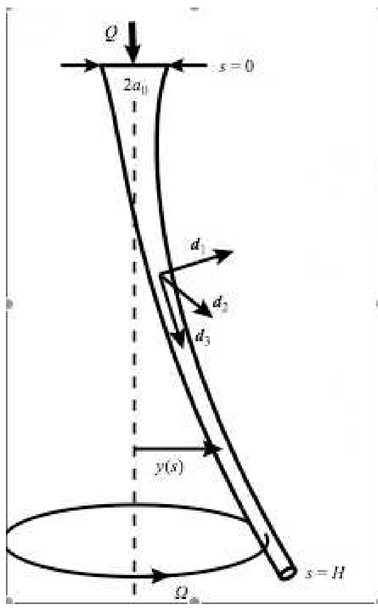

Figure 1: An illustration of the dynamics of the tail that lead to the IG regime. This figure is taken from [4].

In order to understand the multi-valuedness of the coiling frequencies observed in the IG regime, it is necessary to study the dynamics of the longer, tail portion of the viscous flow. The tail is described as the portion of flow that precedes the bend that produces coils. Let s be the arc length of the liquid rope. Ribe has shown that the horizontal displacement, $y(s)$, is a major factor in determining the dynamics of this regime [4]. To achieve steady coiling, the centrifugal force in the end of the tail must be proportional to the gravitational force. Thus,

$$\rho A g \sin\theta \sim \rho A \Omega^2 y. \tag{13}$$

A is cross-sectional area of the liquid rope, assumed to be circular, and θ is the vertical angle with respect to the surface. The displacement y at the end of the tail is close to the radius of the coil, R, and $\sin\theta \sim R/H$ [4]. Thus, the frequency of the coil Ω should be proportional to Ω_{IG}, which by the above approximations and (13) is:

$$\Omega_{IG} = \left(\frac{g}{H}\right)^{1/2}. \tag{14}$$

In [4], Ribe uses a whirling liquid string model to conclude that the distance $y(s)$ satisfies the following boundary value problem:

$$k^{-1}\sin[k(1-\tilde{s})]y'' - y' + \tilde{\Omega}^2 y = 0, \quad y(0) = 0, \;\; y(1) < \infty, \tag{15}$$

where $\tilde{s} = s/H$, and $\tilde{\Omega} = \Omega/\Omega_{IG}$. The dimensionless parameter k satisfies the equation

$$2B\cos^2(k/2) - 3k^2 = 0, \tag{16}$$

where $B = \pi A_0^2 g H^2/(vQ)$ is the Buoyancy number.

Nontrivial solutions satisfying the above equation exist only for particular eigenfrequencies, giving rise to a set of discrete coiling frequency values $\tilde{\Omega}_n$ [4]. Therefore, the observed frequencies, Ω, can be

$$\Omega \approx \tilde{\Omega}_n \cdot \Omega_{IG}. \tag{17}$$

This helps explain the observed jumps in frequency in the IG regime. In fact, Ribe shows that as $k \to \pi$, indicating strong stretching ($a_0 >> a_1$), the observed frequencies satisfy (17).

2 Methods/Experimental Setup

A syringe is filled with approximately 4 ccs of honey and then clamped in place at a set height above a platform. The height of the syringe above the platform can be changed through the use of a mechanical jack attached to the experimental apparatus. On top of the platform is a thin plastic slide, onto which the honey will fall. A 1 kg weight is held above the syringe and then slowly lowered until it contacts the syringe. From then on, the weight will fall, guided through a circular piece of pipe, ensuring that the weight remains centered on the syringe with minimal obstruction. A high speed camera records the fall of the honey from an angle of 30 degrees relative to the plastic slide at a rate of 1000 frames per second. The fall height of the honey is then changed by raising the apparatus onto which the honey filled syringe is clamped. Three trials each are conducted for heights of 4 cm, 7 cm, and 10 cm, and one trial is conducted for heights of 2.4 cm, 5 cm, 8 cm, and 13 cm. Since there are generally close to 80 frames per coil, the coiling frequency is determined with reasonable accuracy by counting the number of frames required for the next coil to be laid down. The radius a_1 can be determined by examining the pixel distance needed to span the diameter of the viscous stream. Axial velocity (U_1) is calculated for each trial by tracking an air bubble or anomaly in the stream for a given number of frames. Volumetric flow rate is then calculated by using the relation $Q = U_1 \pi a_1^2$. Perhaps a more accurate way to measure the volumetric flow rate would include a scale in the experimental setup, in which time sensitive measurements of the mass of honey that has been ejected could be recorded throughout the experiment.

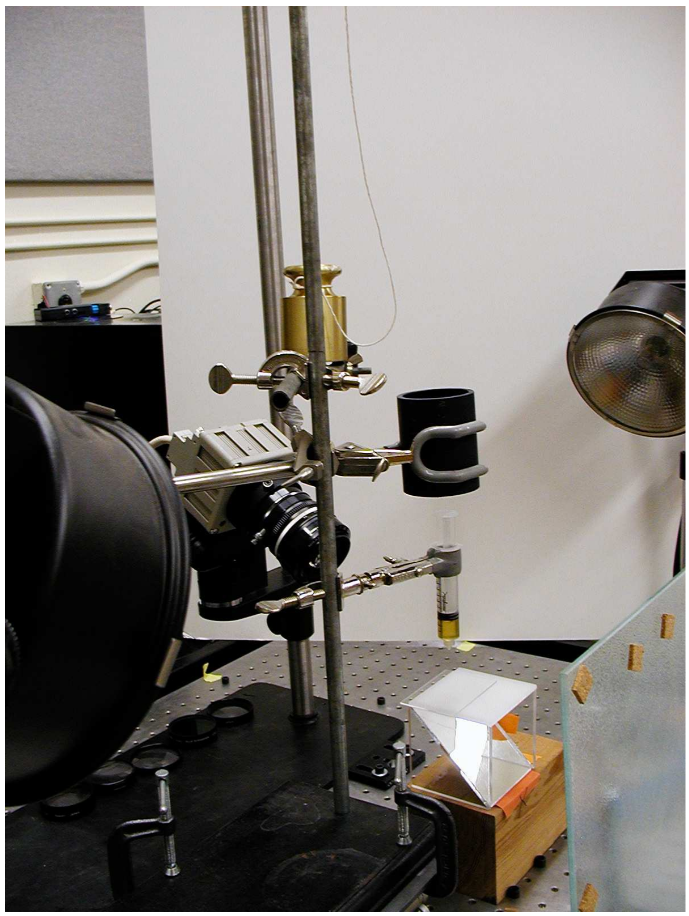

Figure 2: Experimental setup.

3 Results

Experimentally determined coiling frequencies increased with height in all cases, as shown in Figure 3. This has been verified in previous research as well. This occurs because as height increases, viscous stretching leads to a decrease in a_0, causing an increase in the coiling frequency. As the stream width decreases significantly, the inertial and viscous forces grow more rapidly than the gravitational force, thereby progressing towards the inertial regime.

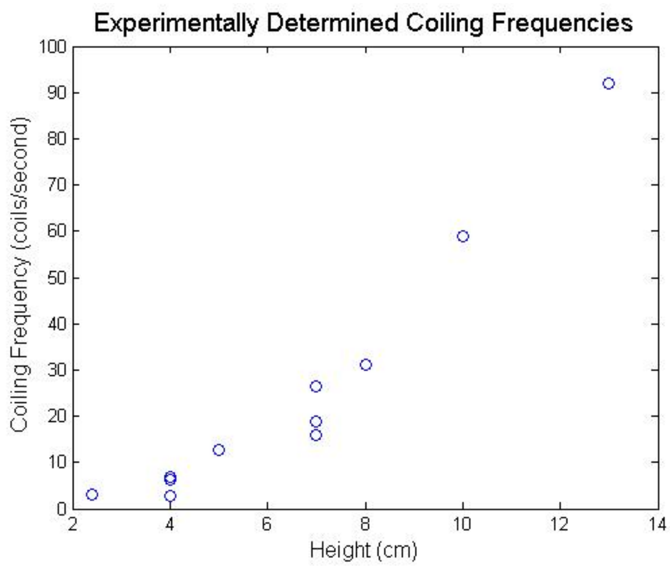

Figure 3: Coiling frequencies versus height.

A single figure of eight has been observed during one of the three experimental trials conducted at a height of 4 cm. The figure of eight resulted in a change in the sense of rotation of the coil and a change in coiling frequency. Following the figure of eight, the honey continued coiling regularly at an approximately steady frequency with no further changes in direction.

The following table summarizes the results:

Height (cm)	Trial	Coiling Frequency ($coils/s$)	Velocity (cm/s)	Flow Rate (cm^3/s)	a_1 (cm)
2.4	1	3.19	3.73	0.0980	0.0915
4.0	1	6.20	9.00	0.1970	0.0835
4.0	2	2.45, 2.72	5.00	0.0560	0.0570
4.0	3	7.00	12.00	0.2900	0.0875
5.0	1	12.70	21.60	0.3180	0.0685
7.0	1	18.73	28.60	0.1767	0.0443
7.0	2	15.79	23.39	0.1906	0.0509
7.0	3	26.49	27.48	0.2121	0.0496
8.0	1	31.00	33.09	0.2500	0.0500
10.0	1	58.82	48.25	0.1640	0.0329
13.0	1	92.00	48.50	0.1250	0.0290

Values for (1) and (12) were calculated for all experimental trials and are plotted against height (Figures 4 and 5).

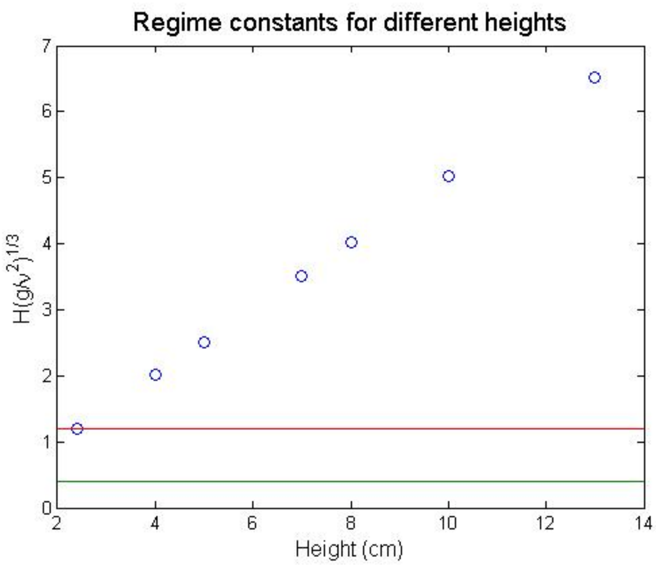

Figure 4: Regime constants $H\left(\frac{g}{\nu^2}\right)^{1/3}$ versus height. The gravitational regime occurs for values $0.08 < H\left(\frac{g}{\nu^2}\right)^{1/3} < 0.4$, the IG regime occurs for values $0.4 < H\left(\frac{g}{\nu^2}\right)^{1/3} < 1.2$ (indicated by horizontal lines), and the inertial regime occurs for values $H\left(\frac{g}{\nu^2}\right)^{1/3} > 1.2$. Circles indicate calculated values of (1) for each experimental trial.

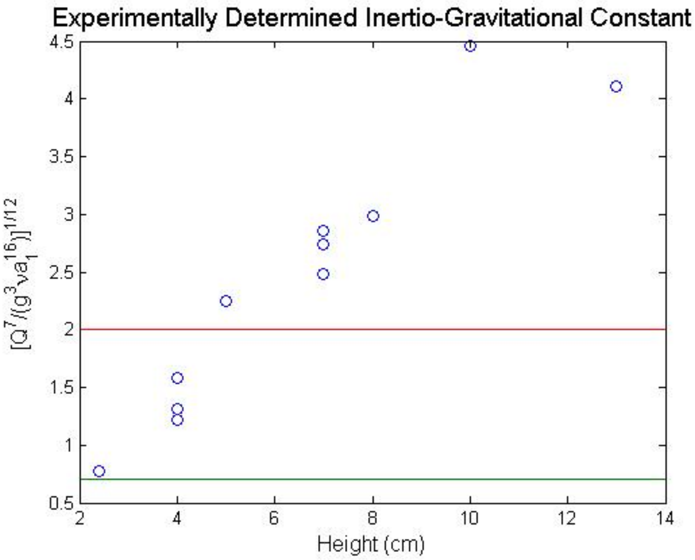

Figure 5: Experimentally determined inertio-gravitational constant $\left(\frac{Q^7}{g^3 \nu a_1^{16}}\right)^{1/12}$ versus height. The IG regime occurs for values $0.8 < \left(\frac{Q^7}{g^3 \nu a_1^{16}}\right)^{1/12} < 2$ (indicated by horizontal lines). Circles indicate calculated values of (12) for each experimental trial.

Notice that there is a disagreement between the two constants on the range of heights that will induce coiling in the IG regime. This is not surprising and is discussed in more detail in the next section. According to Figure 5, although some of the trials were conducted in the IG regime, only one trial exhibited any multi-valuedness.

4 Discussion

Previous studies have shown that in the IG regime, the coiling frequency for a given height is multi-valued [1]. Furthermore, the multi-valued nature of this regime can be characterized more completely by the number of turning points. It has been experimentally verified that Π_1 is directly correlated with the multi-valued nature of $\Omega(H)$ [1].

From Figure 6 it is easy to observe that the number of distinct coiling frequencies associated with a particular height in the IG regime tends to increase for larger values of Π_1. Similarly, it appears that it becomes less common to have multiple coiling frequencies associated with a particular height as Π_1 decreases. Taking into account that flow rates varied between trials in the conducted experiments, it can be determined that for all trials,

$$44 < \Pi_1 < 125. \tag{18}$$

It has been postulated that the true nature of the IG regime can not be observed for $\Pi_1 \leq 300$ [1]. The only observed jump in frequency, accompanied by a figure of eight, occurred during

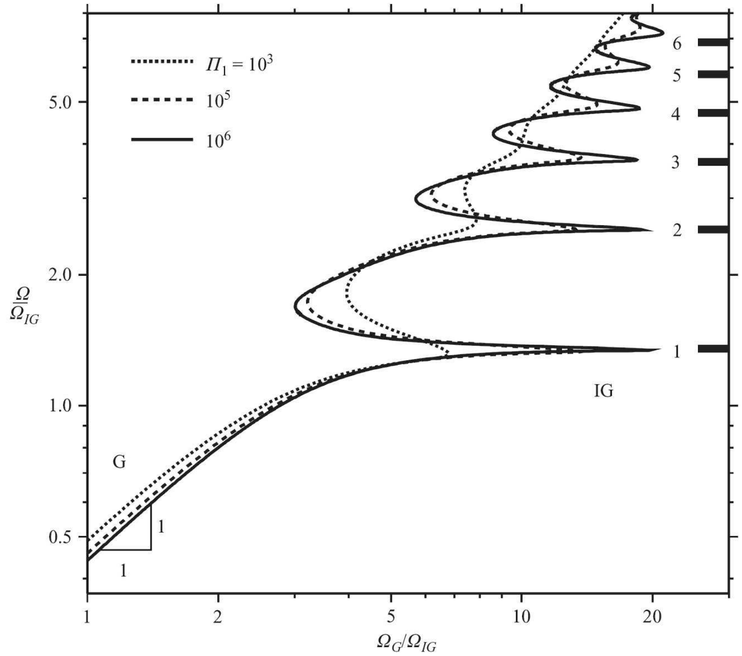

Figure 6: Plot of rescaled frequency for different values of Π_1. As Π_1 increases, the multi-valuedness of $\Omega(H)$ increases. This figure is taken from [1].

the third trial at 4 cm, which had the highest associated Π_1 value. Due to the low viscosity of our honey and a relatively high flow rate, it appears to be extremely difficult to find a range of heights in which multiple coiling frequencies can be observed for a particular height.

In addition, the dimensionless parameter $H(g/\nu^2)^{1/3}$ is not a good indicator for coiling regimes of less viscous liquids. As viscosity decreases, the volumetric flow rate plays a more significant role in determining Π_1. Therefore, perhaps a different dimensionless parameter should be used as an indicator for experiments involving less viscous liquids. It is important to note that as Π_1 increases, the range of heights for the IG Regime increases, and the Inertial regime begins at a later height. This is best seen by looking at $\Omega_I/\Omega_G = \left(\frac{Q^7}{g^3 \nu a_1^{16}}\right)^{1/12}$. As Π_1 increases, the volumetric flow rate is decreasing and viscosity is increasing. Ergo, Ω_I/Ω_G is decreasing and consequently, the dynamics of the system become less inertially dominated. For this reason, we propose another dimensionless parameter to indicate coiling regimes for less viscous liquids:

$$\Xi = \left(\frac{HQ}{a_0^2 \nu}\right)^{1/2}. \tag{19}$$

Future work requires significant testing of Ξ to determine its validity. Figure 7 compares $(HU_1/\nu)^{1/2}$ to F_I/F_G, which are calculated by using observed frequencies. The positive correlation is a good indicator for how sufficient this parameter would be. Moreover, data points that are within $1 < (HU_1/\nu)^{1/2} < 2$ correspond to the same points that are close to the IG regime in Figure 5. We could not test any correlation with Ξ because we did not vary the orifice size a_0.

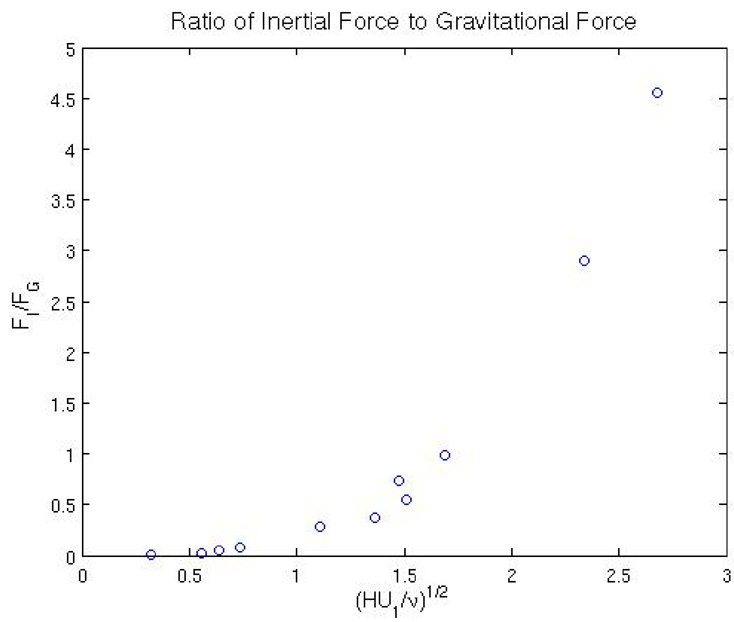

Figure 7: F_I/F_G vs. $(HU_1/\nu)^{1/2}$. Using (4) and (5), forces are calculated using observed data.

5 Conclusion

Experimental data confirms several trends indicated by the scaling laws, including a positive relationship between height and coiling frequency. Several experimental trials were conducted inside of a range of heights that were predicted to induce coiling in the IG regime. However, very little behavior characteristic of coiling in the IG regime was observed. A figure of eight occurred in one trial at a height of 4 cm and accompanied a change of coiling frequency and change in the sense of rotation of the coil. Results suggest that it is hard to observe the distinguishing characteristics of the IG regime, namely multiple coiling frequencies for a fixed height, with high flow rates and low viscosity liquids. This is consistent with previous research, suggesting that a necessary relationship between flow rate and viscosity must be satisfied in order to fully characterize the IG regime.

The possibility of using the dimensionless constant, Ξ, has been suggested as a more reliable way to predict the range of heights for which coiling in the IG regime could occur for less viscous liquids. More trials need to be conducted to verify the validity of this parameter. These experiments should vary the initial orifice size, the viscosity of the liquid being used, and the flow rate.

A more viscous liquid and a lower, more consistent, flow rate would allow one to experimentally study the IG regime in more detail and accuracy, and thus future experiments should devise a setup to allow for lower flow rates and the use of more viscous liquids, such as silicone oil. For such a setup, the value of the dimensionless parameter Π_1 would be larger, and the range of heights needed to observe coiling in the IG regime would be larger as well.

A Appendix

A.1 Determination of Kinematic Viscosity

To ascertain the kinematic viscosity of the honey being used, we conducted a falling ball viscosity test. We filled a graduated cylinder of diameter 2.5 cm with 100 ml of honey and dropped a small ball of diameter .3175 cm. After ensuring that the ball had reached terminal velocity, we recorded how much time was needed for the ball to travel a specified distance, and repeated the experiment multiple times, using stainless steel, brass, and teflon balls of the same size. At terminal velocity, viscous and gravitational forces balance. From Stokes' Law, the following relation is derived to ascertain the viscosity:

$$\nu = \frac{2r^2(\gamma - \rho)g}{9V}, \tag{20}$$

where γ is the density, r is the radius, and V is the observed velocity of the ball. From this relation we were able to calculate the viscosity of the honey to be 88 cm^2/s at 25^o C.

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